

Sometimes it is necessary to express an expression involving radicals using exponents in order to simplify it.

EXAMPLE 5**Representing an expression involving radicals as a single power**

Simplify $\left(\frac{\sqrt[5]{x^8}}{\sqrt{x^3}}\right)^3$.

Albino's Solution

$$\begin{aligned}\left(\frac{\sqrt[5]{x^8}}{\sqrt{x^3}}\right)^3 &= \left(\frac{x^{\frac{8}{5}}}{x^{\frac{3}{2}}}\right)^3 \\ &= \left(x^{\frac{8}{5} - \frac{3}{2}}\right)^3 \\ &= \left(x^{\frac{16}{10} - \frac{15}{10}}\right)^3 \\ &= \left(x^{\frac{1}{10}}\right)^3 \\ &= x^{\frac{3}{10}} \\ &= \sqrt[10]{x^3}\end{aligned}$$

Since this is a fifth root divided by a square root, I couldn't write it as a single radical.

I changed the radical expressions to exponential form and used exponent rules to simplify.

When I got a single power, I converted it to radical form.

In Summary**Key Idea**

- Algebraic expressions involving powers containing integer and rational exponents can be simplified with the use of the exponent rules in the same way numerical expressions can be simplified.

Need to Know

- When evaluating an algebraic expression by substitution, simplify prior to substituting. The answer will be the same if substitution is done prior to simplifying, but the number of calculations will be reduced.
- Algebraic expressions involving radicals can often be simplified by changing the expression into exponential form and applying the rules for exponents.

CHECK Your Understanding

1. Simplify. Express each answer with positive exponents.

- a) $x^4(x^3)$ c) $\frac{m^5}{m^{-3}}$ e) $(y^3)^2$
- b) $(p^{-3})(p)^5$ d) $\frac{a^{-4}}{a^{-2}}$ f) $(k^6)^{-2}$

2. Simplify. Express each answer with positive exponents.

$$\begin{array}{lll} \text{a) } y^{10}(y^4)^{-3} & \text{c) } \frac{(n^{-4})^3}{(n^{-3})^{-4}} & \text{e) } \frac{(x^{-1})^4 x}{x^{-3}} \\ \text{b) } (x^{-3})^{-3}(x^{-1})^5 & \text{d) } \frac{w^4(w^{-3})}{(w^{-2})^{-1}} & \text{f) } \frac{(b^{-7})^2}{b(b^{-5})b^9} \end{array}$$

3. Consider the expression $\frac{x^7(y^2)^3}{x^5y^4}$.

- Substitute $x = -2$ and $y = 3$ into the expression, and evaluate it.
- Simplify the expression. Then substitute the values for x and y to evaluate it.
- Which method seems more efficient?

PRACTISING

4. Simplify. Express answers with positive exponents.

$$\begin{array}{lll} \text{a) } (pq^2)^{-1}(p^3q^3) & \text{c) } \frac{(ab)^{-2}}{b^5} & \text{e) } \frac{(w^2x)^2}{(x^{-1})^2w^3} \\ \text{b) } \left(\frac{x^3}{y}\right)^{-2} & \text{d) } \frac{m^2n^2}{(m^3n^{-2})^2} & \text{f) } \left(\frac{(ab)^{-1}}{a^2b^{-3}}\right)^{-2} \end{array}$$

5. Simplify. Express answers with positive exponents.

$$\begin{array}{lll} \text{a) } (3xy^4)^2(2x^2y)^3 & \text{c) } \frac{(10x)^{-1}y^3}{15x^3y^{-3}} & \text{e) } \frac{p^{-5}(r^3)^2}{(p^2r)^2(p^{-1})^2} \\ \text{b) } \frac{(2a^3)^2}{4ab^2} & \text{d) } \frac{(3m^4n^2)^2}{12m^{-2}n^6} & \text{f) } \left(\frac{(x^3y)^{-1}(x^4y^3)}{(x^2y^{-3})^{-2}}\right)^{-1} \end{array}$$

6. Simplify. Express answers with positive exponents.

$$\begin{array}{lll} \text{a) } (x^4)^{\frac{1}{2}}(x^6)^{-\frac{1}{3}} & \text{c) } \frac{\sqrt{25m^{-12}}}{\sqrt{36m^{10}}} & \text{e) } \left(\frac{(32x^5)^{-2}}{(x^{-1})^{10}}\right)^{0.2} \\ \text{b) } \frac{9(c^8)^{0.5}}{(16c^{12})^{0.25}} & \text{d) } \sqrt[3]{\frac{(10x^3)^2}{(10x^6)^{-1}}} & \text{f) } \frac{\sqrt[10]{1024x^{20}}}{\sqrt[9]{512x^{27}}} \end{array}$$

7. Evaluate each expression. Express answers in rational form with positive exponents.

$$\begin{array}{l} \text{a) } (16x^6y^4)^{\frac{1}{2}} \text{ for } x = 2, y = 1 \\ \text{b) } \frac{(9p^{-2})^{\frac{1}{2}}}{6p^2} \text{ for } p = 3 \\ \text{c) } \frac{(81x^4y^6)^{\frac{1}{2}}}{8(x^9y^3)^{\frac{1}{3}}} \text{ for } x = 10, y = 5 \\ \text{d) } \left(\frac{(25a^4)^{-1}}{(7a^{-2}b)^2}\right)^{\frac{1}{2}} \text{ for } a = 11, b = 10 \end{array}$$

8. Evaluate. Express answers in rational form with positive exponents.

a) $(\sqrt{10\,000x})^{\frac{3}{2}}$ for $x = 16$

b) $\left(\frac{(4x^3)^4}{(x^3)^6}\right)^{-0.5}$ for $x = 5$

c) $(-2a^2b)^{-3}\sqrt{25a^4b^6}$ for $a = 1, b = 2$

d) $\sqrt{\frac{(18m^{-5}n^2)(32m^2n)}{4mn^{-3}}}$ for $m = 10, n = 1$

9. Simplify. Express answers in rational form with positive exponents.

a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$

c) $\left(\frac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}}$

b) $\left(\frac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

d) $\left(\frac{(x^{18})^{\frac{-1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5}$

10. If $M = \frac{(16x^8y^{-4})^{\frac{1}{4}}}{32x^{-2}y^8}$, determine values for x and y so that

T

a) $M = 1$

b) $M > 1$

c) $0 < M < 1$

d) $M < 0$

11. The volume and surface area of a cylinder are given, respectively, by the formulas

A

$$V = \pi r^2 h \quad \text{and} \quad SA = 2\pi rh + 2\pi r^2.$$

a) Determine an expression, in simplified form, that represents the surface area-to-volume ratio for a cylinder.

b) Calculate the ratio for a radius of 0.8 cm and a height of 12 cm.

12. If $x = -2$ and $y = 3$, write the three expressions in order from least to greatest.

$$\frac{y^{-4}(x^2)^{-3}y^{-3}}{x^{-5}(y^{-4})^2}, \frac{x^{-3}(y^{-1})^{-2}}{(x^{-5})(y^4)}, (y^{-5})(x^5)^{-2}(y^2)(x^{-3})^{-4}$$

13. How is simplifying algebraic expressions like simplifying numerical ones?

C

How is it different?

Extending

14. a) The formula for the volume of a sphere of radius r is $V(r) = \frac{4}{3}\pi r^3$. Solve this equation for r . Write two versions, one in radical form and one in exponential form.

b) Determine the radius of a sphere with a volume of $\frac{256\pi}{3} \text{ m}^3$.

15. Simplify $\frac{\sqrt{x(x^{2n+1})}}{\sqrt[3]{x^{3n}}}$, $x > 0$.